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Thermally Induced Response of Flexible Structures: A Method for Analysis

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Introduction

THE phenomenon of a thermally induced spacecraft instability made itself dramatically known during the flight of the OGO-IV spacecraft in the late 1960's. On that flight the 60-ft experiment boom immediately broke into a sustained large amplitude oscillation which severely compromised spacecraft performance. A detailed analysis of the problem can be found in Ref. 1. A less dramatic but still serious thermally induced instability occurred during the flight of Explorer 45 (SSS-A). On this flight, minute oscillations of the four experiment booms, caused by modulations of the thermal input, were phased in such a manner that a nutationally destablizing effect was produced. A detailed analysis of this problem can be found in Ref. 2. In addition to these postflight analyses, the work presented in Refs. 3 and 4 provides the techniques necessary for the preflight analysis of the potentiality of a thermally induced instability.

To go beyond state-of-the-art capabilities presently implies that thermally deformable appendages other than beams be considered. While this problem is of minimal importance in 1979, it will not always be so. Currently, extremely large antennas and platforms are being proposed for space application which will require shape to be controlled so as to cancel out the performance degrading effects of thermal deformation. The stability analysis of proposed shape control systems will require analysis techniques which are not currently available.

The purpose of this paper is to present a generalized approach to the problem of obtaining a computationally practical simulation model for a thermally driven structure. The formulation is such that it can be utilized in a stand-along manner or be meshed into a general multibody simulation model such as that presented in Refs. 5 and 6. As presented, the formulation is restricted to the dynamic range for which linearized radiation is a valid assumption.

Method

Assume a structure complex enough to prohibit thermoelastic continuum analysis. In this situation, finite-element techniques are inevitably used to obtain flexible-body characteristics and either finite-difference or finite-element techniques used to obtain thermal response characteristics. The problem is to dynamically couple the two disciplines so as to obtain closed-loop response information. Furthermore, the problem is compounded by the fact that the analyst requiring the multidiscipline study, say a control system analyst, usually does not have the necessary expertise to set up either a finite-element structural model or a finite-element or finite-difference thermal model. This communications problem can be partially overcome by basing the multidiscipline formulation upon currently existing, widely accessible general purpose analysis programs such as NASTRAN, SPAR, SINDA, TRASYS, DISCOS, etc.

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Structural Analysis

Once the structure under investigation has been adequately defined, the analyst can usually obtain a finite-element model compatible with the preferred in-house structure's program, e.g., NASTRAN, SPAR, STARDYNE. This program will set up the following set of dynamic equations:

$$M\ddot{U}+KU=0 \tag{1}$$

where M=mass matrix, K=stiffness matrix, and U=displacement vector. Then, upon user request, the program will automatically compute a set of the most significant natural frequencies and associated vibration modes. Let N_v = total number of vibration modes computed, ω_n = nth natural frequency ($n=1,2,\ldots N_v$), and φ_n = vibration mode associated with ω_n; and normalize the modes according to the following orthonormalization condition:

$$\varphi_m^T M \varphi_n = M_T \delta_{m,n} \tag{2}$$

where M_T = scalar total mass of structure; $\delta_{m,n}$ = Kronecker Delta function, and the superscript T is used to signify "transpose of matrix."

Thermal Analysis

The most difficult portion of the study will be obtaining the required data from the thermal analyst. The underlying cause of this is the fact that general-purpose thermal analysis programs are structured to determine temperature fluctuations at particular points in the spacecraft over a temperature range which prohibits linearization of fourth-power radiation terms. For this reason, it will be best to belabor the thermal analysis portion of the study.

To start, there are two methods currently used: the finite-difference method and the finite-element method. For the purposes of this study, either approach may be taken.

If the finite-element approach is used, a set of grid points are defined which are then connected by heat-conducting elements. The difficult problem with this approach is specification of the heat-conducting elements used to simulate internal radiation effects. This gets particularly complex when the effects of shadowing must be taken into account. However, once done, the finite-element program is structured to set up the appropriate heat capacitance and conduction matrices and to numerically solve the equations.

If the finite-difference approach is used, a set of node points are defined and a general thermal balance equation is set up for each. The user is required to specify all thermal coupling between node points. While this approach at first glance appears more laborious than the finite-element approach, it provides the analyst with an added degree of flexibility which frequently is required to solve real-world thermal problems, e.g., those which must account for shadowing of internal radiation and reflection.

The "Nodal Network Thermal Balance Program," available through COSMIC[7] provides a convenient framework to build upon. Utilizing notation adapted from the program documentation, let T_I = absolute temperature at node I, T_{I_0} = average mean temperature at node I, and ΔT_I = variation from the mean at node I, such that

$$T_I = T_{I_0} + \Delta T_I \tag{3}$$

COND (I,J) = thermal conductance between nodes I and J
RADI (I,J) = effective emittance multiplied by the shape factor area product between nodes I and J (internal radiation effects)
RADS (I) = effective emittance multiplied by surface area of node I (radiation to space)
ER (I) = solar input at node I
WCP (I) = heat capacity of node I
σ = Stefan-Boltzmann constant

and write the thermal balance equation for node I as

$$\mathrm{WCP}(I)\dot{T}_I = \mathrm{ER}(I) + \sum_J \mathrm{COND}(J,I)[T_J - T_I] + \sigma\left\{\sum_J \mathrm{RADI}(J,I)[T_J^4 - T_I^4] - \mathrm{RADS}(I)T_I^4\right\} \quad (4)$$

Linearize this equation by use of Eq. (3), bring all terms linear in ΔT_I to the left-hand side of the equation, and obtain

$$\mathrm{WCP}(I)\Delta\dot{T}_I + \sum_J \mathrm{COND}(J,I)[\Delta T_I - \Delta T_J] + \sigma\left\{\sum_J 4\,\mathrm{RADI}(J,I)[T_{I_0}^3\Delta T_I - T_{J_0}^3\Delta T_J] + 4\,\mathrm{RADS}(I)T_{I_0}^3\Delta T_I\right\} = \mathrm{ER}(I) + \sum_J \mathrm{COND}(J,I)[T_{J_0} - T_{I_0}] + \sigma\left\{\sum_J \mathrm{RADI}(J,I)[T_{J_0}^4 - T_{I_0}^4] - \mathrm{RADS}(I)T_{I_0}^4\right\} \quad (5)$$

or in condensed matrix notation

$$B\dot{V} + CV = P \quad (6)$$

where B = heat capacitance matrix, C = linearized heat conduction matrix, V = temperature variation matrix, and P = thermal input matrix.

Consider the zero thermal input situation

$$B\dot{V} + CV = 0 \quad (7)$$

Assume a solution of the form

$$V = \psi e^{-\lambda t} \quad (8)$$

and immediately obtain the standard eigen-analysis problem

$$[\lambda 1\, B - C]\psi = 0 \quad (9)$$

where 1 is the unit diagonal matrix.

Let N_T = total number of thermal modes, λ_m = mth natural thermal decay constant ($m = 1,2,...,N_T$), and ψ_m = thermal mode associated with λ_m; and normalize the modes according to the following orthonormalization condition:

$$\psi_m^T B \psi_n = \delta_{m,n} \quad (10)$$

It should be remarked that the adopted terminology "natural thermal decay constant" and "thermal mode" is not textbook standard; however, the obvious semantic correlation between the terms "natural frequency" and "vibration mode" is intentional and useful from a physical interpretation viewpoint.

These eigen-properties are now to be used to obtain a modal solution to the general heat conduction Eq. (6). Assume a solution of the form

$$V = \sum_m q_m(t)\psi_m \quad (11)$$

Substitute into Eq. (6), apply the orthonormality condition (10), and obtain the set of uncoupled generalized thermal coordinate equations:

$$\dot{q}_m(t) + \lambda_m q_m(t) = Q_m \quad (12)$$

where

$Q_m = \psi_m^T P$ = generalized thermal input for the mth generalized thermal coordinate (13)

Thermally Induced Motion

The net effect of time-dependent thermal gradients in a thermally deformable flexible structure is to cause the reference state from which elastic restoring forces are measured to be time-dependent. To account for this fact, Eq. (1) must be modified and rewritten in the form

$$M\ddot{U} + K(U - U_T) = 0 \quad (14)$$

where U_T = thermal equilibrium displacement vector.

This thermal equilibrium displacement vector is a time-dependent quantity which is not only dependent upon the state variables associated with the solution to the general heat conduction Eq. (6), but also upon the thermal deformation characteristics of the structure under investigation.

To affect a solution of the thermally induced motion Eq. (14), one must first develop a computationally expedient means for defining the thermal equilibrium displacement vector as a function of the generalized thermal coordinates. This can be done as follows.

1) Impose a temperature distribution on the structure exactly equal to $T = T_0 + 1\,\psi_m$. That is, the absolute temperature of any point in the structure is equal to the mean temperature at the point plus one times the amplitude of the mth thermal mode. (By definition, the structure is in the zero thermal stress state when it is at T_0.) The details of how this is to be done are a function of the program to be used to compute static thermal deformation.

2) Utilizing NASTRAN or any other suitable program, compute the thermally deformed shape of the structure associated with the above-defined temperature distribution; let $\tilde{U}_m$ represent the thermal equilibrium displacement vector associated with a temperature distribution equal to that defined by the mth thermal mode ψ_m ($m = 1,2,...,N_T$). It is extremely important to note that this is a time-independent quantity which need be computed only once during the data input preparation stage of the analysis.

3) Express the thermal equilibrium displacement vector associated with each thermal mode in a series format based upon the structural modes of vibration. It is a computationally simple task to compute the following scalar quantities.

$$\beta_{m,n} = (1/M_T)\varphi_n^T M \tilde{U}_m \quad (15)$$

so that one may write

$$\tilde{U}_m = \sum_n \beta_{m,n}\varphi_n \quad (16)$$

4) Recall that the solution to the general heat conduction Eq. (6) is given by Eq. (11). It follows from the assumption of linearity and the definition of the thermal equilibrium modal displacement vector $\tilde{U}_m$ that the equation used to define the thermal equilibrium displacement vector at time t is given by

$$U_T = \sum_m q_m(t)\tilde{U}_m \quad (17)$$

or by direct substitution of Eq. (16) into Eq. (17):

$$U_T = \sum_m \sum_n q_m(t) \beta_{m,n} \varphi_n \tag{18}$$

5) Assume a solution to the thermally induced vibration Eq. (14) of the form

$$U = \sum_n a_n(t) \varphi_n \tag{19}$$

Use the eigen-analysis matrix identity

$$\omega_n^2 M \varphi_n = K \varphi_n \tag{20}$$

the orthonormality condition given by Eq. (2), and define

$$\alpha_n(t) = \sum_m \beta_{m,n} q_m(t) \tag{21}$$

Then, by direct substitution into the thermally induced vibration Eq. (14), one immediately obtains the set of uncoupled generalized displacement coordinate equation:

$$\ddot{a}_n(t) + \omega_n^2 [a_n(t) - \alpha_n(t)] = 0 \tag{22}$$

for $n = 1,2,\ldots,N_v$. It should also be obvious to the reader that proportional damping and nonthermal loads can also be treated by a trivial extension of notation.

Thermal Input

Thermally induced instabilities are inevitably caused by a mechanism which modulates the thermal input at a system resonance. Consider the terms on the right-hand side of Eq. (5) and note that all except the solar input term $ER(I)$ are time-independent. In nearly all situations the thermal input is given simply by an a priori function of time; this, however, is not true for the thermally induced vibration problem. In this situation, the dynamic motion of the flexible structure must be accounted for in the specification of the thermal input. In general, this can be done by expressing the thermal input at node point I as

$$P_I = P_I^*(t) + \sum_n \gamma_{I,n} a_n(t) \tag{23}$$

where $\gamma_{I,n}$ is the constant coefficient dependent upon geometry and sun orientation, which defines the component of the thermal input which is a linear function of the nth generalized displacement coordinate at node I, and $P_I^*(t)$ is that portion of the thermal input at node I which is not deformation-dependent.

Summation

It is both useful and instructive to collect the operative equations and to cast them into a compact matrix format. Let

U = column matrix, displacement at all structural grid points
V = column matrix, temperature variation from mean at all thermal node points
a = column matrix, all modal displacements $a_n(t)$, $n = 1,\ldots,N_v$
q = column matrix, all modal temperatures $q_m(t)$, $m = 1,\ldots,N_T$
Φ = rectangular modal matrix, all φ_n, N_v columns
Ψ = rectangular modal matrix, all ψ_m, N_T columns
ω^2 = diagonal matrix, all ω_n^2, $n = 1,\ldots,N_v$
λ = diagonal matrix, all λ_m, $m = 1,\ldots,N_T$
β = rectangular matrix, all $\beta_{m,n}$, $m = 1,\ldots,N_T$; $n = 1,\ldots,N_v$
γ = rectangular matrix, all $\gamma_{I,n}$, N_v columns
P^* = column matrix, all $P_I^*(t)$

Then, from Eqs. (19) and (11), displacement at any grid point and temperature variation at any node point is obtained from

$$U = \Phi a \tag{24}$$

$$V = \Psi q \tag{25}$$

respectively. All generalized displacement and thermal coordinates required by Eqs. (24) and (25) are obtainable by simultaneous solution of Eqs. (12) and (22). Using Eqs. (21) and (23), these equations can be put into the following matrix format:

$$\begin{Bmatrix} \ddot{a} \\ \dot{a} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} 0 & \omega^2 & -\omega^2 \beta^T \\ -1 & 0 & 0 \\ 0 & -\psi^T \gamma & \lambda \end{bmatrix} \begin{Bmatrix} \dot{a} \\ a \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \psi^T P^* \end{Bmatrix} \tag{26}$$

Remarks

The method presented herein is a generalization of the successfully applied methods used in Ref. 1 to study thermally induced vibrations of long thin-walled cylinders of open section. This method assumes the existence of a structural model which can be used to obtain standard clamped free modes and frequencies. It assumes the existence of a thermal model which can be used to obtain temperature profiles. It assumes that the dynamic range of interest is such that thermal radiation effects can be linearized. Finally, it assumes that a static thermal deformation model exists which can be used to compute thermal deformation for particular thermal profiles. The thermal input is arbitrary; however, care must be taken to assure that over the dynamic range of interest, all time-independent radiation coefficients will remain constant.

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